C-7

ANALYSIS OF MULTIPLE DIELECTRIC WAVEGUIDE SYSTEMS WITH EXTENDED POINT-MATCHING METHOD

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Quaternary dielectric waveguide systems, which could be applied to optical fibers and dielectric image lines, are analyzed with an extended point-matching method. Theoretical results are confirmed with microwave experiments.

I. Introduction

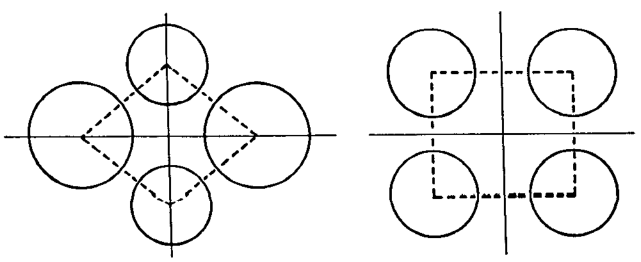
A class of dielectric waveguides including optical fibers and dielectric image lines can be treated from a unified viewpoint. When the cross-section of a dielectric waveguide is composed of a set of homogeneous media, a general solution of the wave equation for each medium is automatically given in many cases. If a method to treat various types of dielectric boundary shape is devised, therefore, it would be possible to solve field problems associated with the dielectric waveguide of this class.

The point-matching method is such a method which was first applied by Goell[1] to the analysis of a single rectangular dielectric waveguide and then, by the authors to other dielectric waveguides[2,3]. With this method, the boundary condition is approximately satisfied at a finite number of points on boundary surface. As a result, boundary value problems of this class are reduced to a set of linear algebraic equations which can be numerically solved on a computer.

We have developed the point-matching method by using multiple coordinates to be adapted to multiple dielectric waveguide systems. The case of two parallel circular dielectric rods has been discussed by Wijngaard[4]. This paper describes the results of the analysis of quaternary dielectric waveguide systems based on the extended point-matching method. It is shown that propagation constants can be obtaind within a practical size of computation time and memory capacity in the case of the rhombic and the rectangular array of dielectric waveguides with circular cross-sections.

II. Quaternary Dielectric Waveguide Systems

Two types of quaternary dielectric waveguide systems to be analyzed here are shown in Fig.1, (a) and (b), that is, the rhombic array and the rectangular array. Because of symmetry, boundary points only on the first quadrant are necessary in the process of analysis. Each dielectric waveguide is assumed to have a nearly circular cross-section.



(a) Rhombic array (b) Rectangular array

Fig.1 Quaternary dielectric waveguide systems.

The wave numbers for the inside and the outside medium of the dielectric waveguide are denoted by k and k_0, respectively. Each dielectric waveguide has its own circular cylindrical coordinates (r_i, θ_i, z; i=1,2,3,4) as shown in Fig.2 to express total electromagnetic fields properly. The use of such multiple coordinate system is simple but very effective in reducing the number of necessary boundary points.

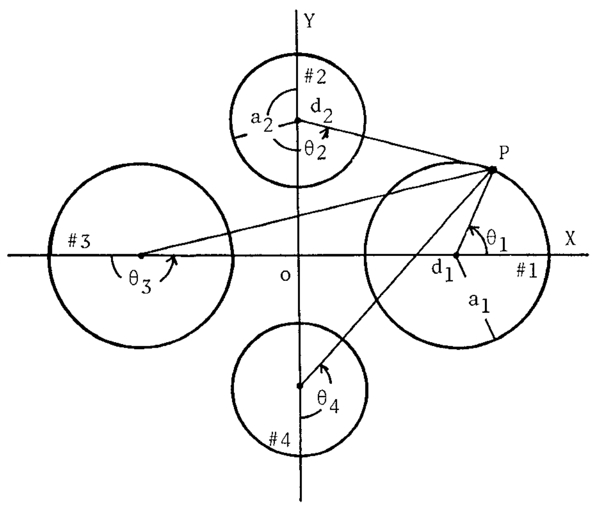


Fig.2 Multiple coordinate system.

III. Analysis Procedure

Electromagnetic fields associated with this system are assumed to have a propagation factor, exp(jωt-jβz). Since the fields can be expressed by a linear combination of functions which satisfy the wave equation, the z-components of the fields are written as

$$E_{zi} = \sum_{n=0}^{N-1} a_{ni} J_n(hr_i) \sin(n\theta_i + \phi_{ni}) \qquad (i=1,2,3,4) \quad (1)$$

$$H_{zi} = \sum_{n=0}^{N-1} b_{ni} J_n(hr_i) \cos(n\theta_i + \phi_{ni}) \qquad (i=1,2,3,4) \quad (2)$$

for the inside medium, and

$$E_{zo} = \sum_{i=1}^{4} \sum_{n=0}^{N-1} c_{ni} K_n(pr_i) \sin(n\theta_i + \phi_{ni}) \quad (3)$$

$$H_{zo} = \sum_{i=1}^{4} \sum_{n=0}^{N-1} d_{ni} K_n(pr_i) \cos(n\theta_i + \phi_{ni}) \quad (4)$$

for the outside medium, where J_n denotes the Bessel's function of the first kind and K_n the modified Bessel's function of the second kind. a_n, b_n, c_n, and d_n are

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constants to be determined.

The existence of hybrid mode solutions has to be assumed for this system. When E_z and H_z are given, remaining field components can be derived from these components. The series expansion of each component has N undetermined constants. There are 4 tangential components on the inside and outside boundary of a dielectric waveguide in the first quadrant. Since there are 8N undetermined constants in all, it is necessary to find 8N equations.

When M matching points are selected on the dielectric boundaries in the first quadrant, each point produces 4 boundary condition equations. Hence, there are 4M equations in all. Therefore, a square matrix equation can be formed by setting 4M=8N. Since the determinant of this matrix should vanish to obtain non-trivial solutions, this determinantal equation leads to the equation of β. The accuracy of calculated values of β from this equation depends on the size of N and the selection of M points. Therefore, N is determined while observing the convergence of β.

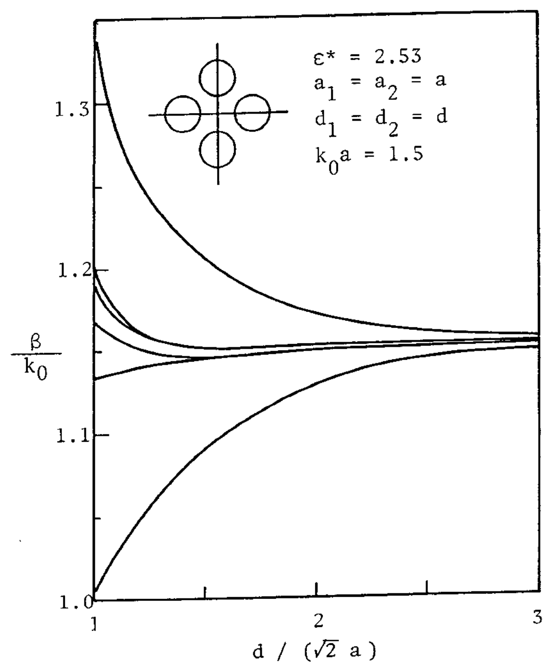

Fig. 3 The propagation constant β vs. the separation d.

IV. Numerical Results

When the number of the matching points, M, is increased, better values of fields can be obtained. On the other hand, the computation time is increased proportionally to M^2. There must be some trade between the accuracy and computation time. The accuracy of β of the dominant mode is still in the order of 10^{-3} even in the extreme case of contacting waveguides in a rhombic array. When the waveguides are separated to the distance d=2a, the accuracy becomes 10^{-7} with N=7 as shown in Fig.3. Convergence gets better in general when frequency is increased. The computation time of β was about 10 s on the HITAC M-170 computer.

The difference of the dielectric constant between the inside and outside of the dielectric waveguide is usually small in the case of optical fibers, and relatively large in the case of millimeter waveguides. However, there was no problem in computing the propagation constant in either case.

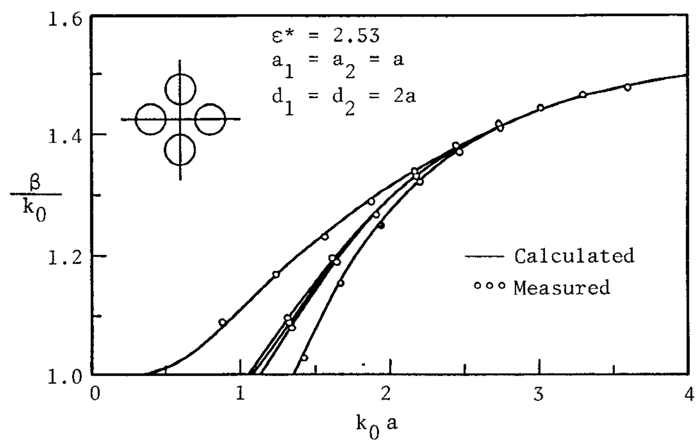

Fig.4 Comparison of calculated values of β / k_0 with measured values. (Same diameter)

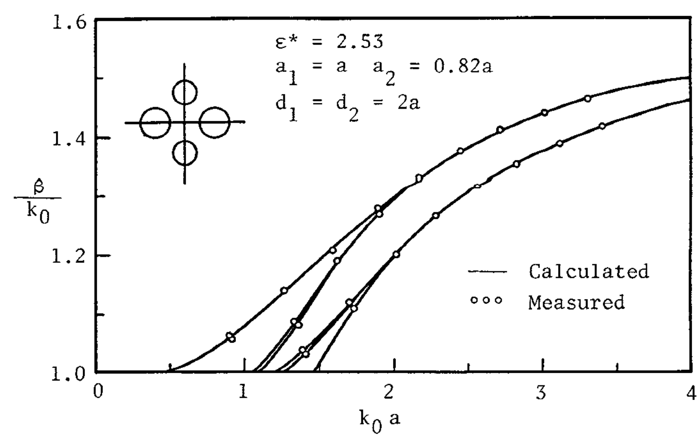

Fig.5 Comparison of calculated values of β / k_0 with measured values. (Different diameter)

V. Experimental Results

A special microwave resonator was made by placing conductor plates at both ends of the quaternary dielectric waveguide system. Each dielectric waveguide was made of Rexolite ($\varepsilon^*=2.53$) rods which have the diameter of 2.22 cm or 2.71 cm, and the length of 8.8 cm. The output from a sweep generator ($f=2\sim18$GHz) was weakly coupled to the resonator. The dispersion characteristics were then plotted by using the resonance frequency and the resonator wavelength.

Some experimental results of the propagation constant are compared with the calculated values in Fig.4 and Fig.5. The agreement between the experimental and calculated values indicates the usefulness of the extended point-matching method in the analysis of multiple dielectric waveguide systems as shown in Fig.1 and Fig.6. Fig.1 shows a model of an optical fiber with high-density core distributions. Fig.6 illustrates coupled dielectric image lines on various conductor plates.

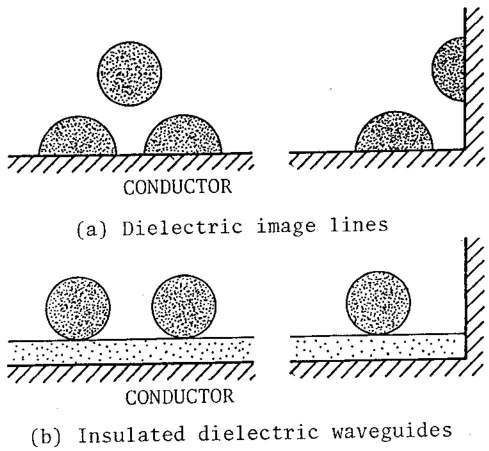

Fig.6 Some applications of quaternary dielectric waveguide systems.

Acknowledgement

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References

1. J. E. Goell, "A circular-harmonic computer analysis of rectangular dielectric waveguides," Bell Syst. Tech. J., vol. 48, pp. 2133-2160, Sept. 1969.

2. E. Yamashita, K. Atsuki, O. Hashimoto, and K. Kamijo, "Modal analysis of homogeneous optical fibers with deformed boundaries," IEEE Trans. Microwave Theory Tech., vol. MTT-27, pp.352-356, Apr. 1979.

3. E. Yamashita, K. Atsuki, and R. Kuzuya, "Composite Dielectric Waveguides," IEEE Trans. Microwave Theory Tech., vol. MTT-28, pp. 986-990, Sept. 1980.

4. W. Wijngaard "Guided normal modes of two parallel circular dielectric rods," Jour. Opt. Soc. Am., vol. 63, pp. 944-950, Aug. 1973.